Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

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To cite this Article Sartory, W. K.(1970) 'Fractional Cleanout in a Continuous-Flow Centrifuge', Separation Science and Technology, 5: 2, 137 — 143
To link to this Article: DOI: 10.1080/00372367008057955
URL: http://dx.doi.org/10.1080/00372367008057955

SEPARATION SCIENCE, 5(2), pp. 137–143, April, 1970

Fractional Cleanout in a Continuous-Flow Centrifuge

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Summary

A theoretical equation is derived for the fraction of particles removed from the feed stream of a continuous-flow centrifuge for the case of an arbitrary velocity profile in the centrifuge. A definition of the fractional cleanout is adopted which is somewhat different from that used in earlier work, and it leads to a simpler final equation.

INTRODUCTION

In order to avoid pelleting of the separated particles, continuous-flow centrifuges are often operated by first filling the centrifuge with a density gradient and then continuously flowing a light feed stream containing the particles to be separated axially along the length of the rotor (*1*). Centrifugal buoyant forces due to the heavy gradient solution confine the feed stream to a thin layer near the core (see Fig. 1). Otherwise the gradient plays no part in the present analysis. The particles sediment from the feed layer to the outlying gradient as the feed stream moves along the core, and can eventually be banded in the gradient. The present work is concerned with the theoretical prediction of the fraction of particles removed from the feed stream as

* NIH-AEC Molecular Anatomy Program is jointly sponsored by the National Cancer Institute, the National Institute of General Medical Sciences, the National Institute of Allergy and Infectious Diseases, and the U.S. Atomic Energy Commission. The Oak Ridge National Laboratory is operated by Union Carbide Corporation for the U.S. Atomic Energy Commission.

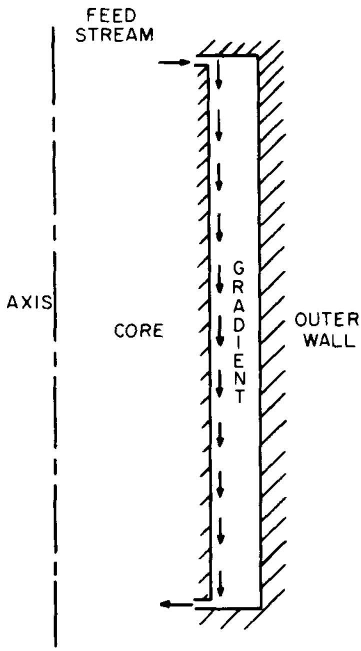

FIG. 1. Sketch of centrifuge rotor.

a function of the feed rate and other parameters describing the centrifuge and the particles.

Berman (*2*) has considered the prediction of the fractional cleanout in a continuous flow rotor without the density gradient. In such a case the feed stream fills the entire space between the core and outer wall of the rotor, and the radial distribution of the axial velocity of the feed stream is known. In the present analysis, the manner in which the axial velocity drops to zero in the gradient is not obvious, and the distribution of axial velocity is not assumed to be known.

THEORY

The solution contains a solvent, a macromolecular or particulate species, and a gradient-forming solute. Let c_i be the concentration of the ith species in g/cc of solution, and $\bar{U}_i$ be the vector velocity of the ith species. Then conservation of the ith species requires that

$$\partial c_i/\partial t + \text{div}\,(c_i\bar{U}_i) = 0 \tag{1}$$

where t is time. If $\bar{v}_i$ is the partial specific volume of the ith species in cc/g, then

$$\sum_i c_i\bar{v}_i = 1.0 \tag{2}$$

If we assume that the $\bar{v}_i$ are constants, by multiplying Eq. (1) by $\bar{v}_i$ and summing over i we get

$$\text{div}\ \bar{U} = 0 \tag{3}$$

where $\bar{U} = \Sigma_i c_i \bar{v}_i \bar{U}_i$ is the volume-mean velocity of the fluid.

Let the radial and axial components of the volume-mean velocity in the centrifuge rotor be given by $u(r,z)$ and $w(r,z)$, where r,z are the radial and axial coordinates. We assume that u and w are independent of the transverse coordinate θ, and that the transverse component of velocity is zero. Equation (3) becomes

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Since the flow of the feed stream is confined to a thin layer near the core, we have $r \approx r_1$, the core radius, and approximately

$$\partial u/\partial r + \partial w/\partial r = 0 \tag{5}$$

We can then introduce a stream function $\psi(r,z)$ such that

$$w = \partial\Psi/\partial r; \qquad u = -\partial\Psi/\partial z \tag{6}$$

Equations (6) define ψ only up to an additive constant. Since u vanishes at the core wall, ψ is a constant along the core so we choose $\psi(r_1,z) = 0$. Then $\psi(r,z)$ is the volumetric flow rate of fluid, per unit of perimeter, which passes between the core wall and the point (r,z).

The macromolecular species is carried along with the fluid but also sediments radially. The radial and axial mass flux of macromolecules are then given by

$$J_r = (u + s\omega^2 r)c = (-\partial\Psi/\partial z + s\omega^2 r)c \tag{7}$$

$$J_z = wc = (\partial\Psi/\partial r)c \tag{8}$$

where c is the concentration, s is the sedimentation coefficient of macromolecules, and ω is the angular velocity of the rotor. Again, assuming that the flow is confined to a thin layer near the core, we replace r by r_1 and get

$$J_r = (-\partial\Psi/\partial z + s\omega^2 r_1)c \tag{9}$$

In steady state,* conservation of macromolecules requires that the divergence of the mass flux vanish, and again using the thin-layer approximation we have

$$\partial J_r/\partial r + \partial J_z/\partial z = 0 \tag{10}$$

Substituting Eqs. (8) and (9) into Eq. (10), assuming s to be a constant, and simplifying, we have

$$(-\partial\Psi/\partial z + s\omega^2 r_1)\partial c/\partial r + (\partial\Psi/\partial r)\partial c/\partial z = 0 \tag{11}$$

We now define a modified stream function

$$\Psi_1 = \Psi - s\omega^2 r_1 z \tag{12}$$

Then

$$(-\partial\Psi_1/\partial z)\partial c/\partial r + (\partial\Psi_1/\partial r)\partial c/\partial z = 0 \tag{13}$$

Equation (13) states that the Jacobian of the transformation

$$\left.\begin{aligned} c &= c(r, z) \\ \Psi_1 &= \Psi_1(r, z) \end{aligned}\right\} \tag{14}$$

vanishes identically. It follows that there is a functional dependence of c on ψ_1:

$$c(r, z) = F[\Psi_1(r, z)] \tag{15}$$

which, for arbitrary F, is the general solution of Eq. (13).

As a boundary condition, we require that the radial flux of macromolecules vanish at the impermeable core wall

$$J_r(r_1, z) = 0, \qquad z \geq 0 \tag{16}$$

which, in view of Eq. (7), gives

$$c(r_1, z) = 0, \qquad z \geq 0 \tag{17}$$

We also require that, at the inlet axial position, $z = 0$, the concentration have its initial value c_0

$$c(r, 0) = c_0, \qquad r > r_1 \tag{18}$$

The solution satisfying Eqs. (17) and (18) is

$$c = c_0 S(\Psi_1) \tag{19}$$

where S is the step function:

* Of course, only the feed layer reaches steady state. Macromolecules continue to accumulate in the gradient.

$$S(x) = \begin{cases} 0, & x \leq 0 \\ 1.0, & x > 0 \end{cases} \tag{20}$$

Equation (19) can be written

$$c = c_0 S(\Psi - s\omega^2 r_1 z) \tag{21}$$

THE FRACTIONAL CLEANOUT

The fractional cleanout of macromolecules is defined by

$$f = \frac{\text{influx} - \text{outflux}}{\text{influx}} \tag{22}$$

The feed layer is assumed to be confined to the interval from r_1 to $r_1 + \delta$, $\delta \ll r_1$. The mass influx of macromolecules into the feed layer is given by

$$\begin{aligned} \text{influx} &= 2\pi r_1 \int_{r_1}^{r_1+\delta} J_z|_{z=0}\, dr = 2\pi r_1 \int_{r_1}^{r_1+\delta} [c(r, z)w(r, z)]_{z=0}\, dr \\ &= 2\pi r_1 \int_{r_1}^{r_1+\delta} [c(\partial\Psi/\partial r)]_{z=0}\, dr = 2\pi_1 \int_0^{\Psi_\delta} c|_{z=0}\, d\Psi \end{aligned}$$

where ψ_δ is the value of ψ at the outer edge of the feed layer.

$$\text{influx} = 2\pi r_1 c_0 \int_0^{\Psi_\delta} S(\Psi - s\omega^2 r_1 z)|_{z=0}\, d\Psi = 2\pi r_1 c_0 \Psi_\delta \tag{23}$$

Similarly, the outflux of macromolecules from the feed layer to the exit channels is given by

$$\begin{aligned} \text{outflux} &= 2\pi r_1 \int_{r_1}^{r_1+\delta} J_z|_{z=L}\, dr = 2\pi_1 c_0 \int_0^{\Psi_\delta} S(\Psi - s\omega^2 r_1 z)|_{z=L}\, d\Psi \\ &= 2\pi r_1 c_0 \int_{s\omega^2 r_1 L}^{\Psi_\delta} d\Psi = 2\pi r_1 c_0 (\Psi_\delta - s\omega^2 r_1 L) \end{aligned} \tag{24}$$

where L is the length of the rotor.

The fractional cleanout is then given by

$$f = \frac{s\omega^2 r_1 L}{\Psi_\delta} \tag{25}$$

Since the entire feed stream passes between r_1 and $r_1 + \delta$,

$$\psi_\delta = \int_{r_1}^{r_1+\delta} w(r, z)\, dr \tag{26}$$

is the total volumetric throughput of fluid per unit of perimeter. Using the inner perimeter for a thin layer

$$\psi_\delta = \frac{Q}{2\pi r_1} \tag{27}$$

where Q is the volumetric throughput in cm^3/sec. The fractional cleanout is then given by

$$f = \frac{s\omega^2 2\pi r_1^2 L}{Q} \tag{28}$$

where all quantities are expressed in cgs units.

If the flow rate is expressed as Q_1 in liters/hr, the rotor speed is given in rpm, and the sedimentation coefficient is expressed as s_1 in Svedberg units, Eq. (28) becomes

$$f = \frac{8\pi^3 r_1^2 L s_1}{10^{16}} \frac{(\text{rpm})^2}{Q_1}. \tag{29}$$

Note that s or s_1 is the value of the sedimentation coefficient at the feed stream composition and temperature, and not necessarily the standard value in water at 20°C.

DISCUSSION OF RESULTS

The important result of the present work is that Eq. (29) is obtained independently of the actual shape of the velocity profile in the centrifuge rotor. Equation (29) compares with Eq. (47) of Berman (*2*).

The present work imposes one additional restriction not used by Berman; namely, that the flow occupy a thin layer near the core, but is more general in that it applies to an arbitrary velocity profile. In comparing the present results with those of Berman for a centrifuge radius ratio near 1.0, however, a further discrepancy was found. This discrepancy is due to a difference in the definition of the fractional cleanout. Berman defines f for a batch (nonflow) system and then applies it to a flow system. The present f is defined for a flow system and is believed to be preferable.

The present theory, and that of Berman (*2*) for a radius ratio near 1, predict that complete cleanout ($f = 1.0$) will be reached at the same finite feed flow rate. On the other hand, the experimental results of Perardi, Leffler, and Anderson (*3*) seem to indicate that as the feed rate is reduced, the fractional cleanout approaches asymptotically, without ever quite reaching, the value 1.0. The question then arises as to what might cause any possible error in the present theory. One possible source of error would be channeling. Because of a slight misalignment, or possibly because of Coriolis forces, the flow might not form an even layer around the rotor. The flow would then depend on θ as well as r and z. Such channeling would certainly reduce the cleanout. Back-diffusion of the macromolecules, not accounted for in the present theory, could also reduce the cleanout and, in fact, would theoretically prevent complete cleanout from ever being reached. Finally, turbulence or other convective mixing would greatly aggrevate the back-diffusion problem, and therefore also reduce the cleanout.

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Received by editor December 3, 1969